Analysis 1

31 January 2024
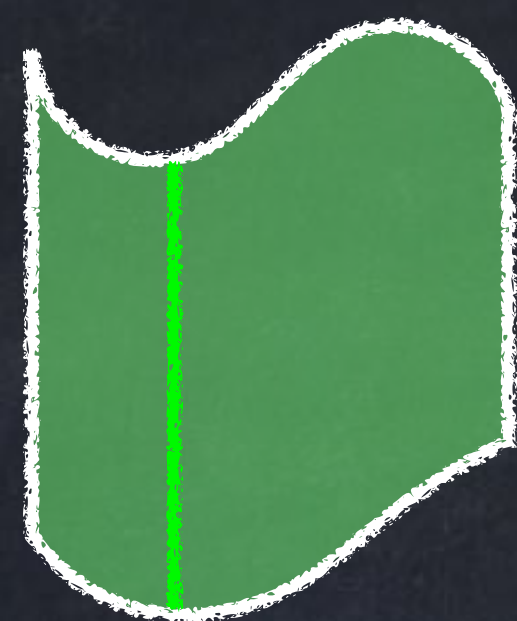
Warm-up: When do the curves $x = y^2$ and $x - 5y = 6$ intersect?

$y^2 - 5y = 6 \quad \rightarrow \quad y = -1, y = 6$

Area integrals

The area of a shape with $a \leq x \leq b$ and with curves on the top and bottom is

$$\int_a^b \Big(\mathrm{Top}(x) - \mathrm{Bottom}(x) \Big) \mathrm{d}x.$$

The area of a shape with $c \leq y \leq d$ and with curves on the left and right is

$$\int_c^d \Big(\mathrm{Right}(y) - \mathrm{Left}(y) \Big) \mathrm{d}y.$$

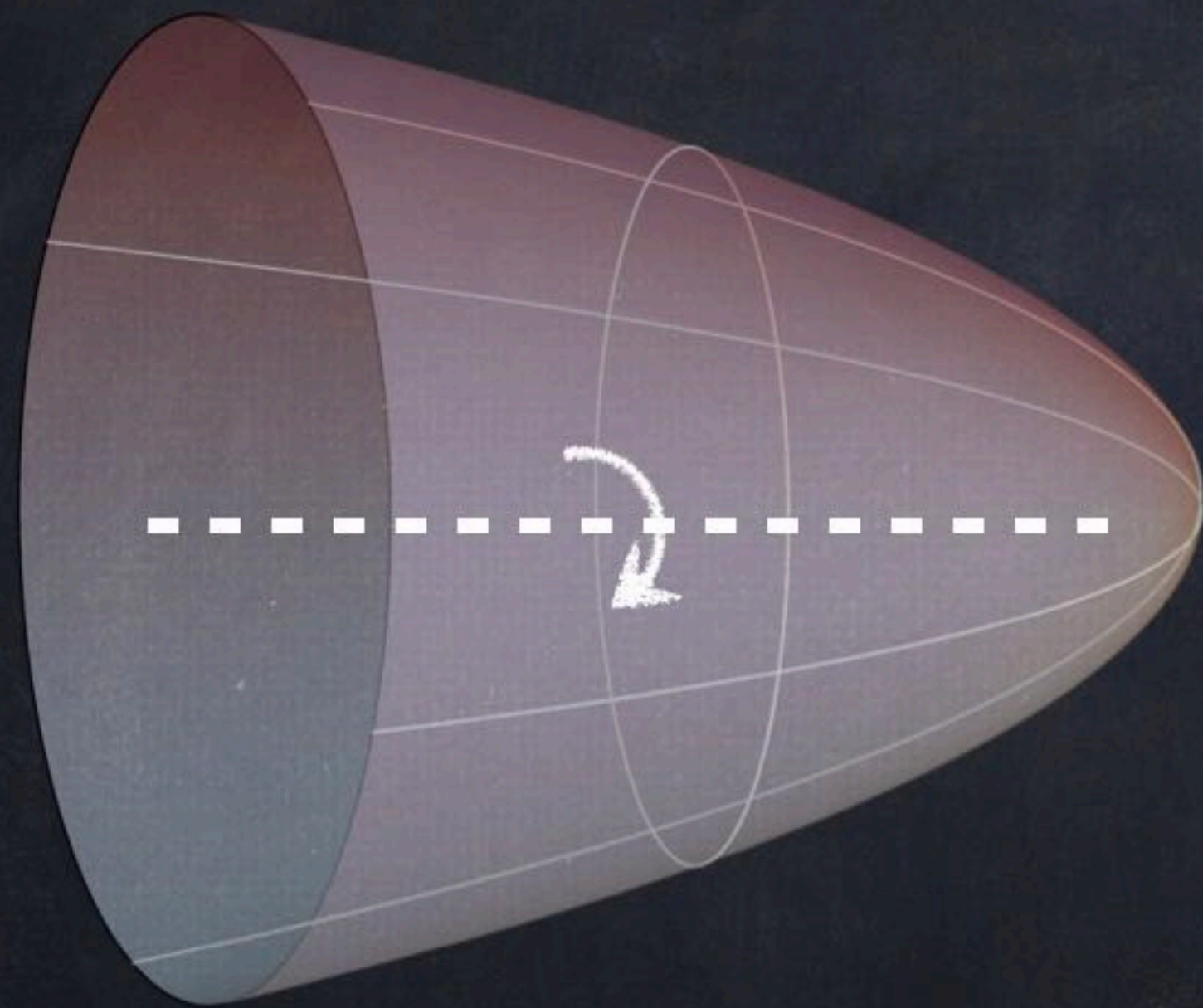For some shapes, both methods are possible.

Example: Compute the area of the region bounded by $x = y^2$ and $x - 5y = 6$.

$$\text{Area} = \int_{-1}^{6} (\text{right} - \text{left})\,dy = \int_{-1}^{6} ((5y+6) - y^2)\,dy = \boxed{\dfrac{343}{6}}$$
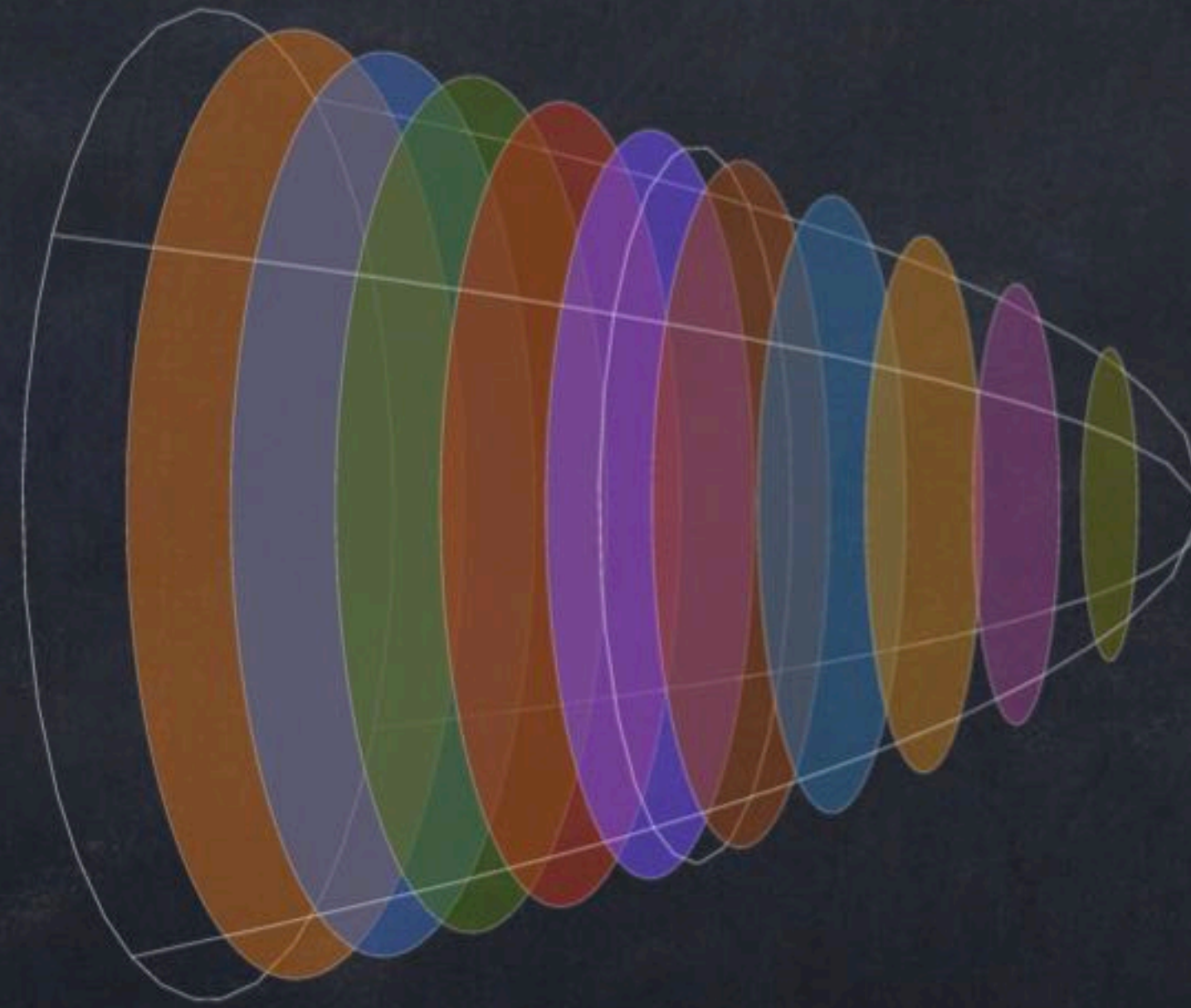
Volume integrals

Area is the integral of height, and *volume is the integral of area*.

$$V = \int_a^b \pi\, r^2 \, \mathrm{d}x$$

$$V = \int_a^b \pi\left(f(x)\right)^2 \mathrm{d}x$$

Region rotated
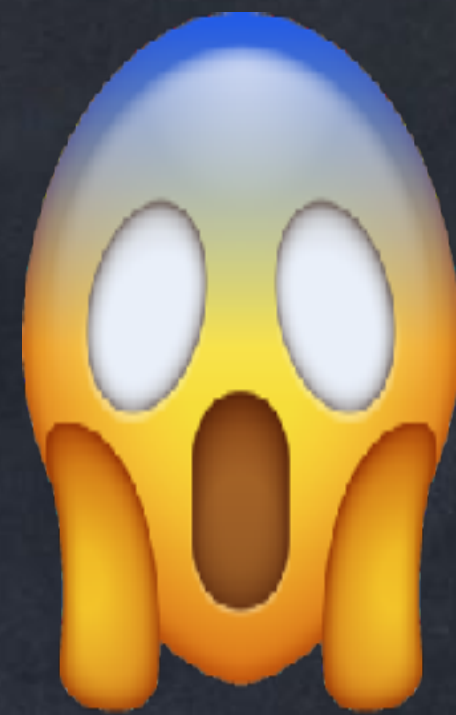around x-axis

Disk area
depends on x

Task: Find the volume of the solid formed by rotating

$$\{(x, y) : 0 \leq x \leq \tfrac{\pi}{4}, 0 \leq y \leq \sin(x)\sqrt{\cos(x)}\}$$

around the x-axis.

Answer: $\pi \dfrac{1}{3}\left(\dfrac{1}{\sqrt{2}}\right)^3 = \dfrac{\pi}{6\sqrt{2}}$

"Bonus topics" activity: discussion of some interesting problems involving sequence limits, derivatives, or integrals.

Celebration of Knowledge 2

The **final exam** is

Wednesday 7 February at 12:00 noon

room 201 / C-1

and a second attempt one week later.

Topics can include…

- Limits (including L'Hospital's Rule)
- Derivative rules
- Tangent lines

- Critical pts, min, max
- Inflection points, concavity
- Taylor polynomials

- Define and indefinite integrals (including substitution, parts)
- Area
- Volume